Mortality Trend – Lee-Carter Model and Analysis

Sept 23, 2011

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Overview

1. Methods to model Mortality Improvement
2. Lee Carter Model
3. Model fit
4. Analysis
5. Result
6. Insurance Industry Experience

Methods Used for Mortality Projection

1. Lee-Carter
2. SOA
3. CMIB
4. Other Actuarial Method

Methods Used for Mortality Projection

SOA Methods –Formula

$$q_x^{1994+n} = q_x^{1994} \cdot (1 - AA_x)^n$$

This method will have a base rate q^{1994} and the reduction factor AA_x for each age x.

SOA Group Annuity Valuation Table Task Force GAR94 Tables

AA_x was obtained:

- 1. Data Source

  CSRS for age 25-65 for 1987-93

  add SSA for age 1-24 and 60-120 for 1977-93
- 2. Average Trends

  Linear Regression of $\log(m_{x,t})$ 5-year age group for data CSRS and SSA for1987-1993 and 1977-1993 respectively

Methods Used for Mortality Projection

CMIB Methods –Formula

$$RF(x,t) = \alpha(x) + [1-\alpha(x)] \cdot [1- f_n(x)]^{t/n}$$

$$q_{x,t} = q_{x,0} \cdot RF(x,t)$$

RF(*x,t*) – Exponential Decay Characterized by two age-dependent parameters. α(x) denotes the value to be asymptotically approached when t ends to infinite, while f_n is the percentage of the total fall (1– α(x)) assumed to occur in n years.

Two set of tables 80 and 92 series (1979-82 and 1991-94 experiences, respectively) for annuitants and pensioners.

- 1. 80 Series – n was fixed at value of 20 and f_{20} at 0.6 for all ages. And α(x) is expressed as:

$$\alpha(x) = \begin{cases} 0.5, & x < 60 \\ \dfrac{x-10}{100}, & 60 \le x \le 110 \\ 1, & x > 110 \end{cases}$$

Methods Used for Mortality Projection

CMIB Methods –Formula

$$RF(x,t) = \alpha(x) + [1-\alpha(x)] \cdot [1 - f_n(x)]^{t/n}$$

$$q_{x,t} = q_{x,0} \cdot RF(x,t)$$

- 2. “92” Series – n remained fixed at 20 but f_{20} values linearly from 0.45 to 0.71 between ages 60 to 110, below 60 and above 110, constant values with the values already mentioned apply. And α(x) and fn are expressed as the following:

$$\alpha(x) = \begin{cases} 0.13, & x < 60 \\ 1 + 0.87 \cdot \dfrac{x-110}{50}, & 60 \le x \le 110 \\ 1, & x > 110 \end{cases}$$

$$f_{20}(x) = \begin{cases} 0.55, & x < 60 \\ \dfrac{(110-x)\cdot 0.55 + (x-60)\cdot 0.29}{50}, & 60 \le x \le 110 \\ 0.29, & x > 110 \end{cases}$$

Methods Used for Mortality Projection

Other Actuarial Method

- Specifications

1. Define Factor
2. Determine “standard”rate for trend
   a) Ratio of the mortality to the previous year’s mortality
   b) Most Recent Mortality Rate
   c) Mean
3. Methods
   a) Arithmetic average
   b) Mean
   c) Geometric average
   d) Weighted average

Lee-Carter Model

1. Data Source
2. Lee-Carter Model
3. Model Analysis

Mortality Trend – Data Source

Region	Source	CalYr	# of CalYr	ProjPeriod	# of ProjYr
Japan	HMD	1947-08	62	1979-08	30
Taiwan	HMD	1970-08	39	1970-08	39
USA	HMD	1933-06	74	1977-06	30
Korea	Korean Population Statistics	1983-03	21	1983-03	21

HMD: Human Mortality Database is a population data that it is administrated by UC Berkley

Mortality Trend – Methodology overview

Lee-Carter's Extrapolating Method

1. Project age pattern and to measure uncertainty
2. SVD to Solve the age-specific parameters as well as the preliminary mortality index
3. ARIMA (0,1,0) a random walk with a drift to project mortality index

Mortality Trend – Lee-Carter Method

General Model

$$\ln(m_{x,t}) = a_x + b_x \cdot k_t + \varepsilon_{x,t} \qquad (1)$$

Where, $m_{x,t}$ is the central mortality rate for age x for year t; a_x and b_x are parameters dependent only on age x; k_t is factor to be modeled as a time series; and the $\varepsilon_{x,t}$ error term, is assumed to have mean zero and standard deviation σ_ε .

Mortality Trend – Lee-Carter Method

Re-Write Model as

$$m_{x,t} = e^{a_x + b_x k_t} \qquad (2)$$

Where, e^{a_x} is the general shape across age of the mortality schedule; the b_x profile tells us which rates decline rapidly and which rates decline slowly in response to changes in k_t ($\frac{d \ln m_{x,t}}{dt} = b_x \cdot \frac{dk_t}{dt}$)

Mortality Trend – Lee-Carter Method

The model (1) cannot be fitted by simple regression methods;

It allows for several solutions.

To deal the above,

Use SVD, and

b_x and k_t is normalized to sum to unity and to zero respectively.

Lee-Carter Method – SVD overview

- SVD Let A be order nxm, then there are unitary matrices U and V, of order n and m respectively, such that $A = UFV^*$, where F is a rectangular diagonal matrix of order mxn,
- $$F = \begin{bmatrix} \mu_1 & & & 0 & \\ & \ddots & & & \\ & & \mu_r & & \\ 0 & & & 0 & \\ & & & & \ddots \end{bmatrix}$$
- With $F_{ii} = \mu_i$. The numbers μ_i are called the singular values of A. They are all real and positive, and they can be arranged so that
$$\mu_1 \geq \mu_2 \geq \cdots \geq \mu_r > 0$$
- Where r is the rank of the matrix.
- V* is a conjugate transpose.

Mortality Trend –Lee-Carter Method

SVD applied to $\ln(m_{x,t})$,

$$\ln(m_{x,t}) = U_{n\times n} F V'_{T\times T} \qquad (3)$$

Or

$$\ln(m_{x,t}) = \sum_{i=1}^{r} \mu_i \cdot u_i(x) \cdot v_i(t) \qquad (4)$$

Note that, V (and also U) is real number, so the conjugate transpose V* is equal to the transpose V'.

Mortality Trend –Lee-Carter Method

Singular values

	Male				Female		
SV	Japan	Taiwan	US	**SV**	Japan	Taiwan	US
μ_1	189.985	147.231	195.399	μ_1	206.401	161.933	214.087
μ_2	6.329	3.600	5.912	μ_2	5.858	2.766	4.483
μ_3	2.758	2.092	2.341	μ_3	3.831	1.490	1.956
μ_4	1.168	1.262	1.090	μ_4	1.166	0.765	0.946
μ_5	0.687	0.672	0.806	μ_5	0.780	0.639	0.672
μ_6	0.626	0.517	0.549	μ_6	0.462	0.521	0.526
μ_7	0.561	0.389	0.428	μ_7	0.388	0.443	0.367
μ_8	0.428	0.342	0.382	μ_8	0.289	0.368	0.322
μ_9	0.327	0.268	0.286	μ_9	0.238	0.311	0.227
μ_{10}	0.271	0.264	0.197	μ_{10}	0.205	0.254	0.219

The first singular value is larger for all three countries (Japan, Taiwan, and US)

Mortality Trend –Lee-Carter Method

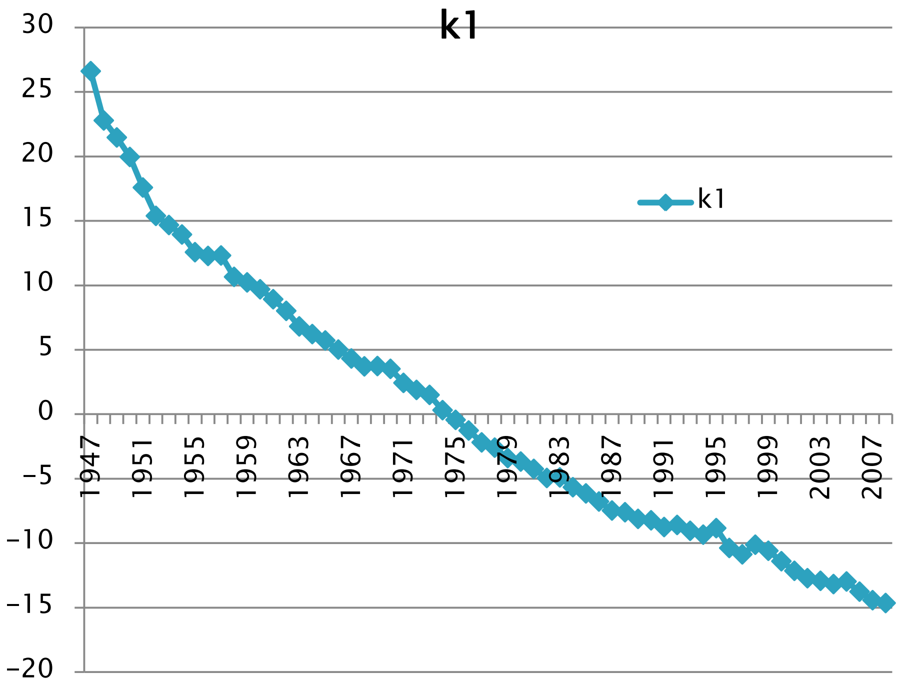
Use the first singular value component

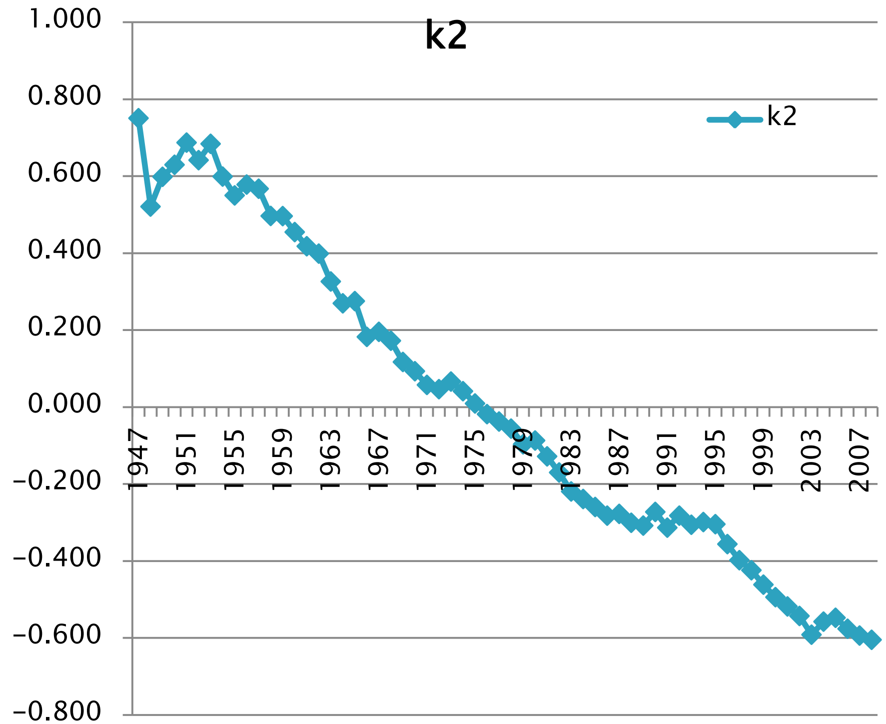
$$\ln(m_{x,t}) = \mu_1 u_1(x) \cdot v_1(t) + \varepsilon_{x,t} \tag{5}$$

The portion of the total temporal variances explained by the first SV component is all over 99%, that seemed captured important of data.

Male			Female		
Japan	Taiwan	US	Japan	Taiwan	US
0.9986	0.9991	0.9989	0.9988	0.9995	0.9994

Mortality Trend –Lee-Carter Method





Mortality Trend –Lee-Carter Method

SVD to Solve $\{a_x\}$, $\{b_x\}$ and $\{k_t\}$,

$$\ln(m_{x,t}) \approx A_x B_t = a_x + b_x \cdot k_t \qquad (6)$$

$$a_x = \frac{1}{T}\sum_t A_x B_t = A_x \left(\frac{\sum_t B_t}{T} \right)$$

$$k_t = \left(\sum_x A_x \right) \cdot \left(B_t - \frac{\sum_t B_t}{T} \right) \qquad (7)$$

$$b_x = \frac{A_x B_t - a_x}{k_t} = \frac{A_x}{\sum_x A_x}$$

Mortality Trend –Lee-Carter Method

Japanese Results – $\{a_x\}$ and $\{b_x\}$ from equation (7) above,

	a_x		b_x	
Age	Male	Female	Male	Female
5-9	-7.7546	-8.1038	0.0764	0.0729
10-14	-8.0693	-8.4369	0.0795	0.0759
20-24	-6.7032	-7.3715	0.0661	0.0663
30-34	-6.5213	-6.9828	0.0643	0.0628
40-44	-5.8181	-6.3220	0.0573	0.0569
50-54	-4.9314	-5.5066	0.0486	0.0496
60-64	-4.0309	-4.6935	0.0397	0.0422
70-74	-3.0759	-3.6493	0.0303	0.0328
80-84	-2.0926	-2.4771	0.0206	0.0223
90-94	-1.2439	-1.4442	0.0123	0.0130

Mortality Trend –Lee-Carter Method

A second stage of estimation of k_t, whereby the k_t's are recalculated from the equation,

$$D(t) = \sum [N(x,t) \cdot \exp(\hat{a}_x + k_t \cdot \hat{b}_x)] \qquad (8)$$

Taking the estimated $\{a_x\}$ and $\{b_x\}$ as fixed from equation (7).

Note that: there is no closed form solution for equation (8) above. (Newton method is employed)

Mortality Trend –Lee-Carter Method

Take an initial k_{t1} equation along with $\{a_x\}$ and $\{b_x\}$ from equation (7) above, the following vector is employed for the Newton's method to obtained the 2nd stage k

Re-write the equation (8) above,

$$F(T) = D(T) - N(X,T)' \cdot e^{a(X)+k(T)\cdot b(X)} \qquad (9)$$

The Jacobian matrix for equation (8) is,

$$J(T) = -N(X,T) \times b(X) \times e^{a(X)+k(T)\times b(X)} \Delta k(T)$$

The first order Taylor series becomes:

$$D(T_i) - N(X,T_i)' \cdot e^{a(X)+k(T_i)\cdot B(X)} + \left(-N(X,T_i) \times b(X) \times e^{a(X)+k(T_{ii})\times b(X)}\right) \cdot \left(k(T_{i+1}) - k(T_i)\right) \to 0$$

Mortality Trend –Lee–Carter Method

ARIMA(0,1,0) time series model that a random walk with drift is found to be a good fit, for the mortality index k_t, That is,

$$k_t = c + k_{t-1} + u_t \qquad (10)$$

Mortality Trend –Lee-Carter Method

From the k_t formula (10),

$$
\begin{aligned}
k_t - k_{t-1} &= C + u_t \\
k_{t-1} - k_{t-2} &= C + u_{t-1} \\
&\vdots \\
k_{t-(m-1)} - k_{t-m} &= C + u_{t-(m-1)}
\end{aligned}
$$

Summing up the above,

$$C \approx \frac{1}{m} \sum_{j=0}^{m-1} \left(k_{t-j} - k_{t-j-1} \right)$$

Mortality Trend –Lee-Carter Method

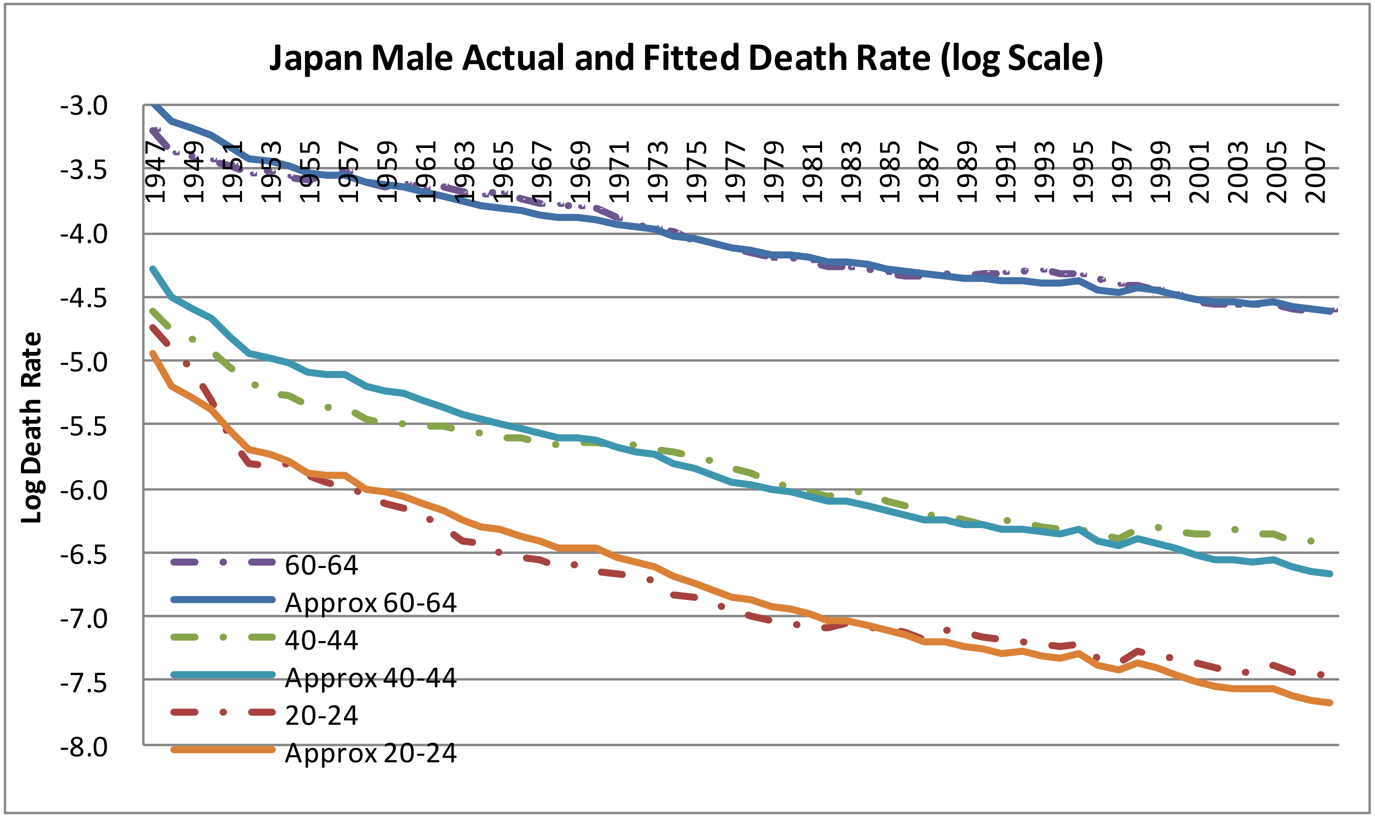
the projection of the k_t into the i years from the current year t,

$$k_{t+i} = k_t + iC + \sum_{j=0}^{i-1} u_{t+j} \qquad (11)$$

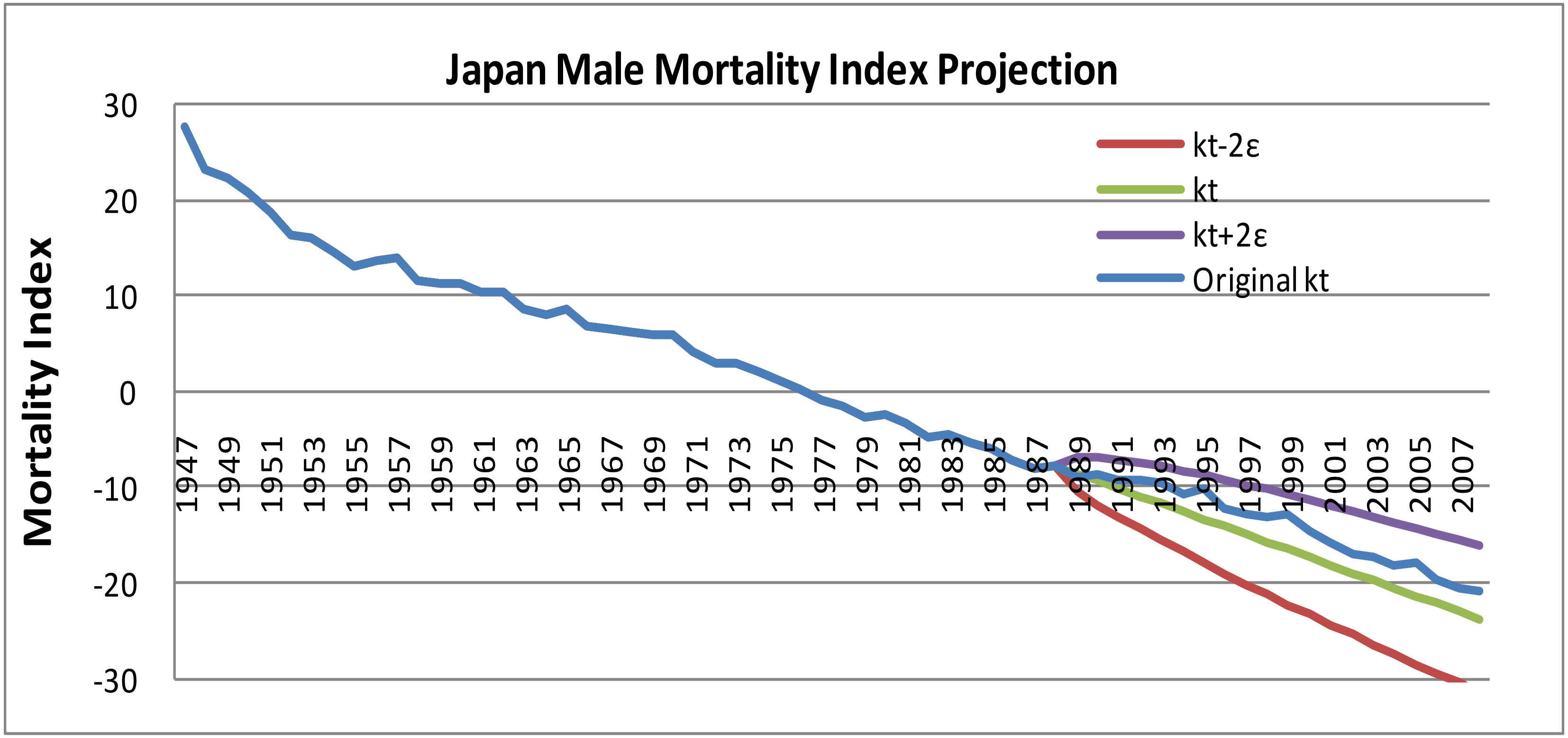
- Equation (11) implies, the quantity for the error term is,

$$\sqrt{\sum_{j=0}^{i-1} u_{t+j}} = \sqrt{i} \cdot \sigma$$

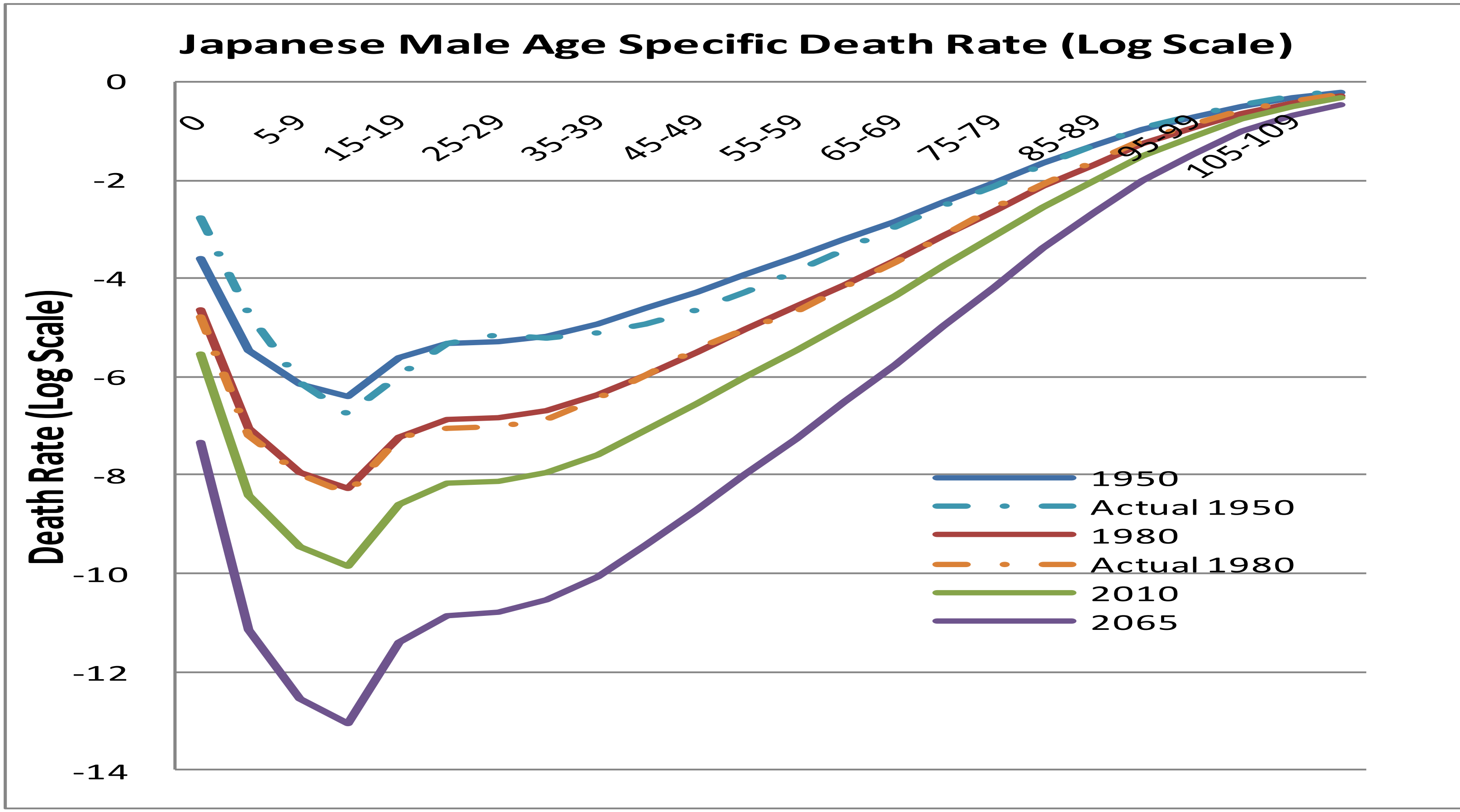
Lee-Carter Model fit Example – Japan Male



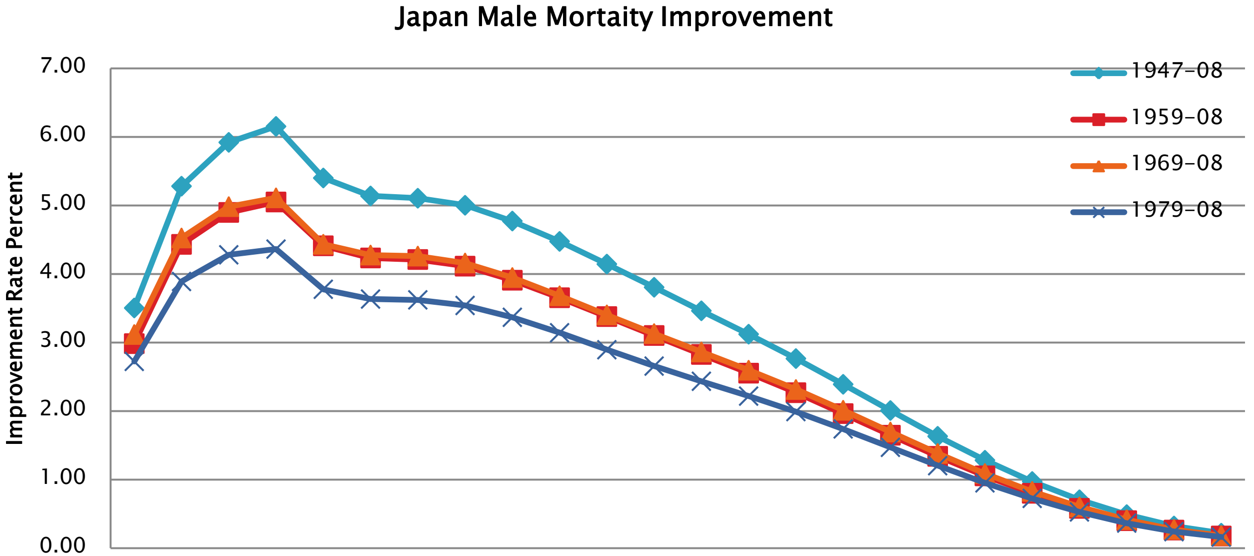
Lee-Carter Model fit Example – Japan Male



Lee-Carter Model fit Example – Japan Male



Lee-Carter Model fit Example – Japan Male



Mortality Trend – Results

Male Mortality Improvement:

Age	Japan	Korea	Taiwan	US
5-9	4.2796	5.7942	3.6728	3.6358
10-14	4.3628	5.9951	3.6694	3.5389
20-24	3.6342	5.1451	3.0832	2.8210
30-34	3.5409	4.7926	2.9008	2.7574
40-44	3.1431	4.1296	2.5833	2.5018
50-54	2.6571	3.5264	2.2444	2.1466
60-64	2.2193	2.9120	1.8646	1.7650
70-74	1.7409	2.2291	1.4328	1.3987
80-84	1.2045	1.3757	1.0185	1.0226
90-94	0.7275	1.3757	0.6749	0.6493

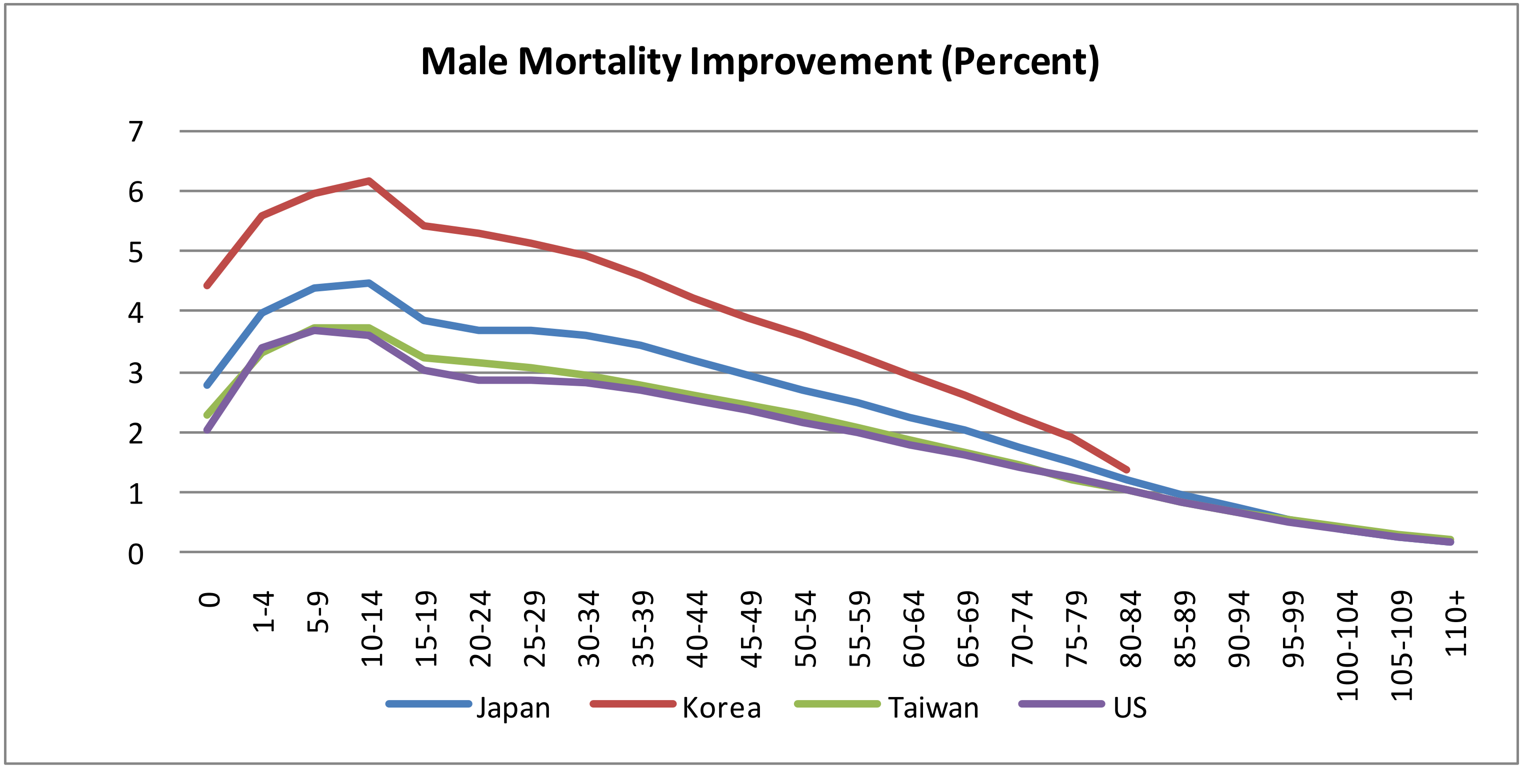
Mortality Trend – Results

Female Mortality Improvement:

	Female			
Age	Japan	Korea	Taiwan	US
5-9	6.7519	6.2529	5.0796	2.3192
10-14	6.9245	6.4956	5.1193	2.3124
20-24	6.1387	5.9135	4.6062	2.0374
30-34	5.8112	5.6422	4.3711	1.9228
40-44	5.2287	5.0969	3.9312	1.7092
50-54	4.5808	4.4627	3.3956	1.4742
60-64	3.9889	3.7685	2.8224	1.2354
70-74	3.2025	2.8585	2.1477	1.0044
80-84	2.2278	1.6831	1.4991	0.7437
90-94	1.3185	1.6831	0.9380	0.4654

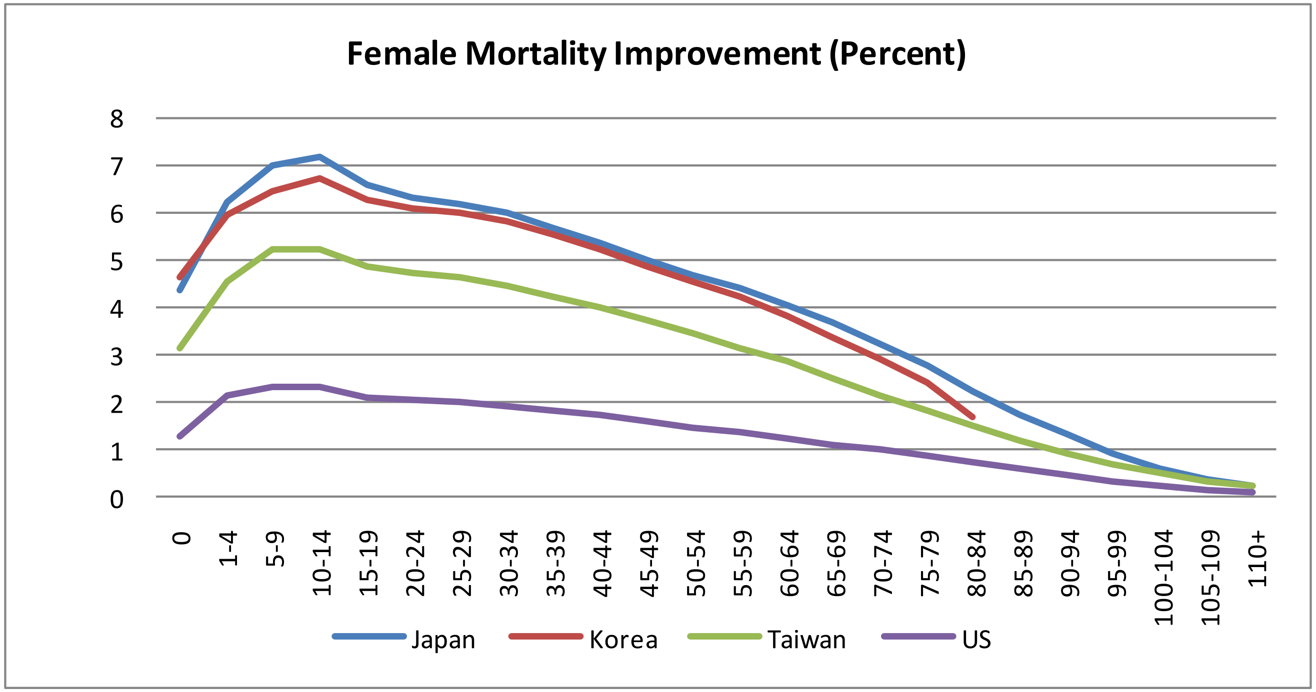
Mortality Trend – Results

Male Improvement:



Mortality Trend – Results

Female Improvement:



Short Term Mortality Trend

a. Leading Cause of Death
b. Mortality Trend for cause of death Data
c. Mortality Rates change by calendar year, type of underwriting, age, sex, Cause of Death etc.

Short Term Mortality Trend

a. Japan: mortality and % of the number of deaths by cause were provided by age, sex, and type of underwriting for each year from 2001 to 05. The mortality and the number of death for leading cause of death by sex and type of underwriting for years from 1992 to 2005.
b. Korea: Mortality Rates by age for leading cause of day by age and sex; and the mortality for study period
c. Taiwan: mortality by sex and medical examination from year 1982 to 2000.

Leading Cause of Death

Leading Cause of Death and the additional Data Used in this analysis

	Leading Cause of death		Study Period	
Region	Current	Prior	Current	Prior
Japan	Neoplasm	Neoplasm	2001-05	1997-98
Korea	Neoplasm	Accident	2000-02	1996-98
China	Accident	N/A	2000-03	N/A
Taiwan	N/A	Accident	N/A	1995-99

Cause of Death - Japan

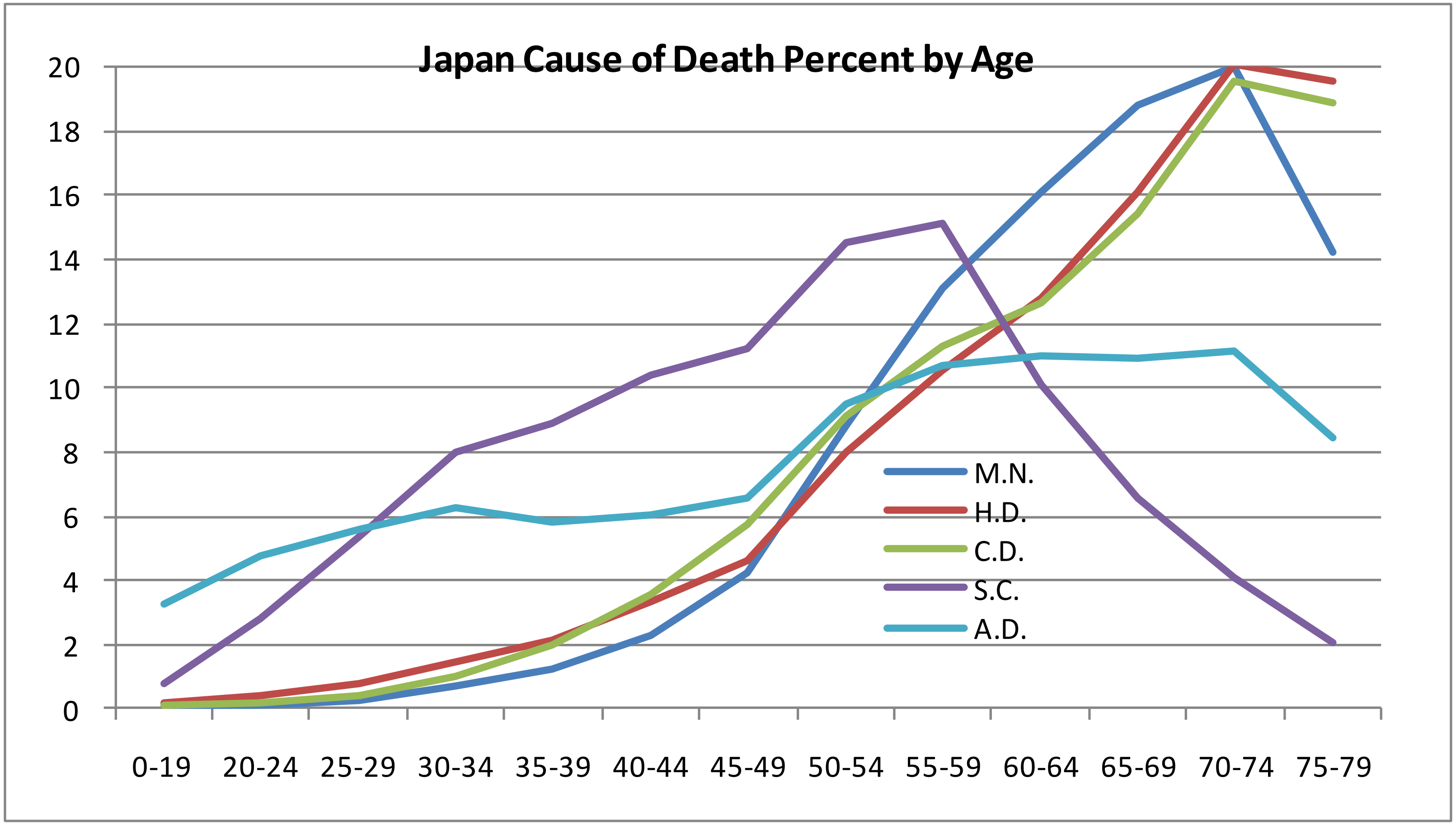
Leading Cause of Death

2001-05	Male		Female		Combined	
Cause	Med	NonMed	Med	NonMed	Male	Female
Malignant Neoplasm	42.61	36.14	47.52	45.62	41.70	47.06
Heard Disease	10.39	11.26	9.05	8.61	10.51	8.94
Cerebrovascular	6.89	7.36	8.54	8.71	6.95	8.58
Suicide	7.00	8.53	4.34	5.64	7.22	4.65
Accident	4.39	6.55	3.26	4.41	4.70	3.54
Subtotal	71.28	69.84	72.72	72.99	71.08	72.78

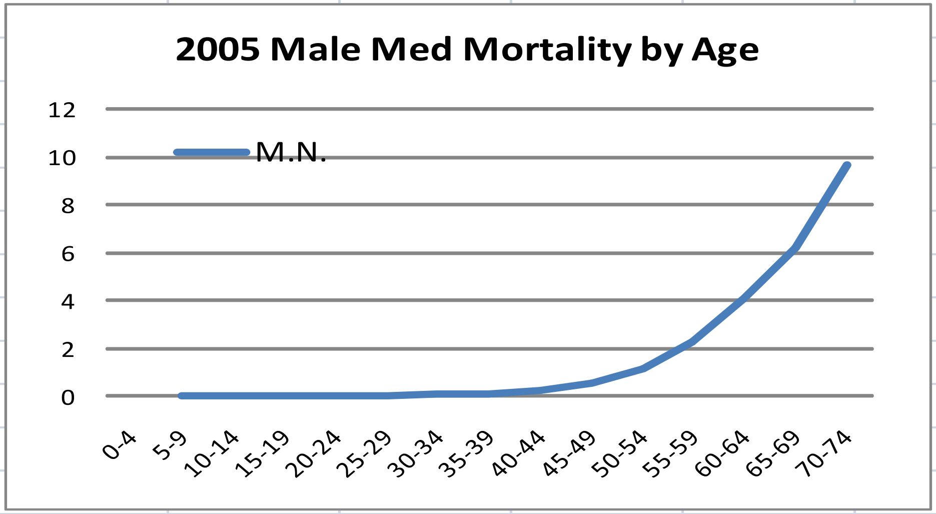
1997-98	Male		Female		Combined	
Cause	Med	NonMed	Med	NonMed	Male	Female
Malignant Neoplasm	41.28	38.51	44.40	42.21	40.61	43.53
Heard Disease	11.93	13.86	10.85	12.17	12.40	11.38
Cerebrovascular	8.18	9.17	10.66	11.13	8.42	10.85
Suicide	5.39	4.20	3.95	4.68	5.10	4.24
Accident	6.60	6.52	4.49	4.55	6.58	4.52
Subtotal	73.38	72.26	74.35	74.74	73.11	74.52

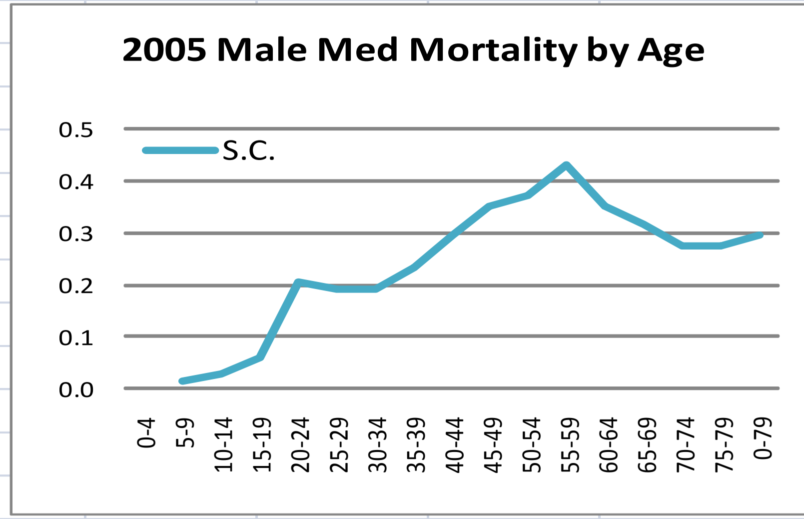
Cause of Death - Japan

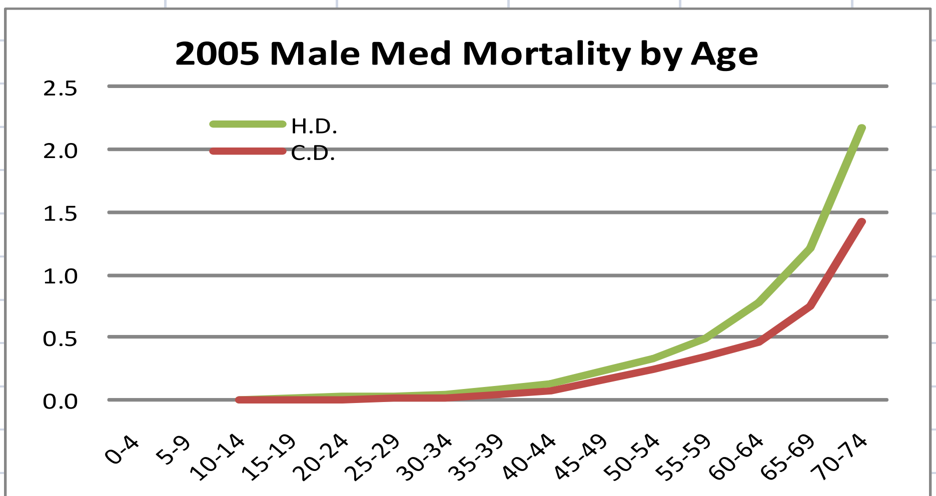
Number of Death Age distribution

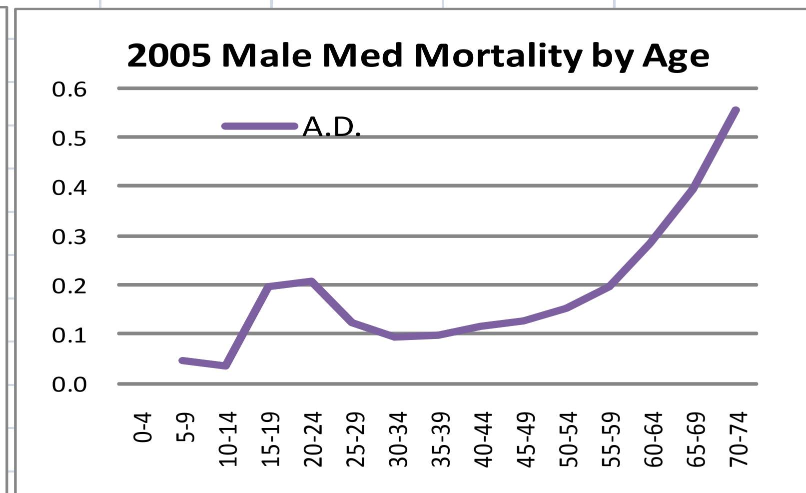


Japan Cause of Death – Mortality Per 1,000









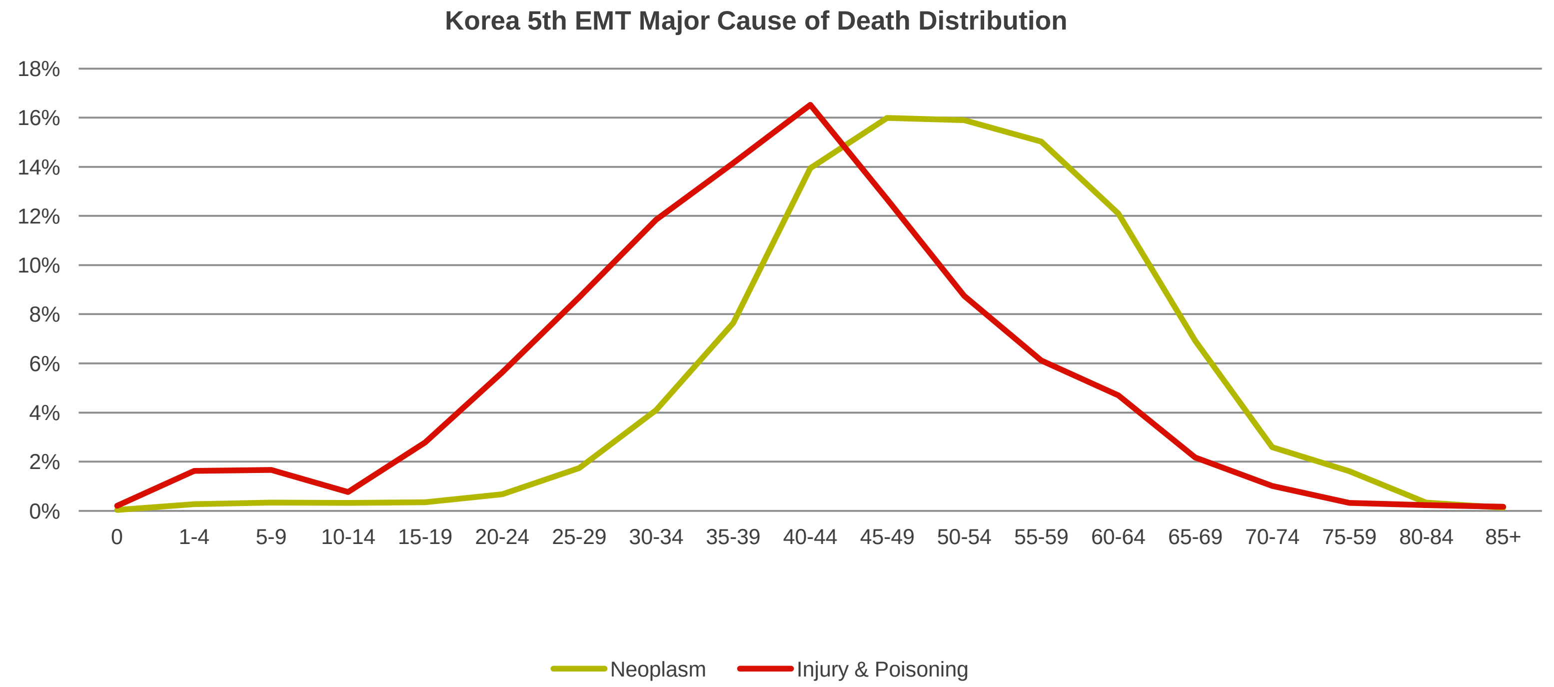
Cause of Death - Korea

Leading Cause of Death

	Male		Female		Combined	
Cause of Death	5thEMT	4thEMT	5thEMT	4thEMT	5thEMT	4thEMT
Neoplasm	29.9	21.5	28.8	28.6	32.4	23.7
Injury & Poisoning	29.1	34.3	14.8	23.6	26.7	31
Circulatory	19.3	16	21.2	19.5	20.2	17.1
Digestive	8.7	9.7	7.8	4.7	8.7	8.1
Subtotal	87	81.5	72.6	76.4	88	79.9

Cause of Death - Korea

Number of Death Age Distribution

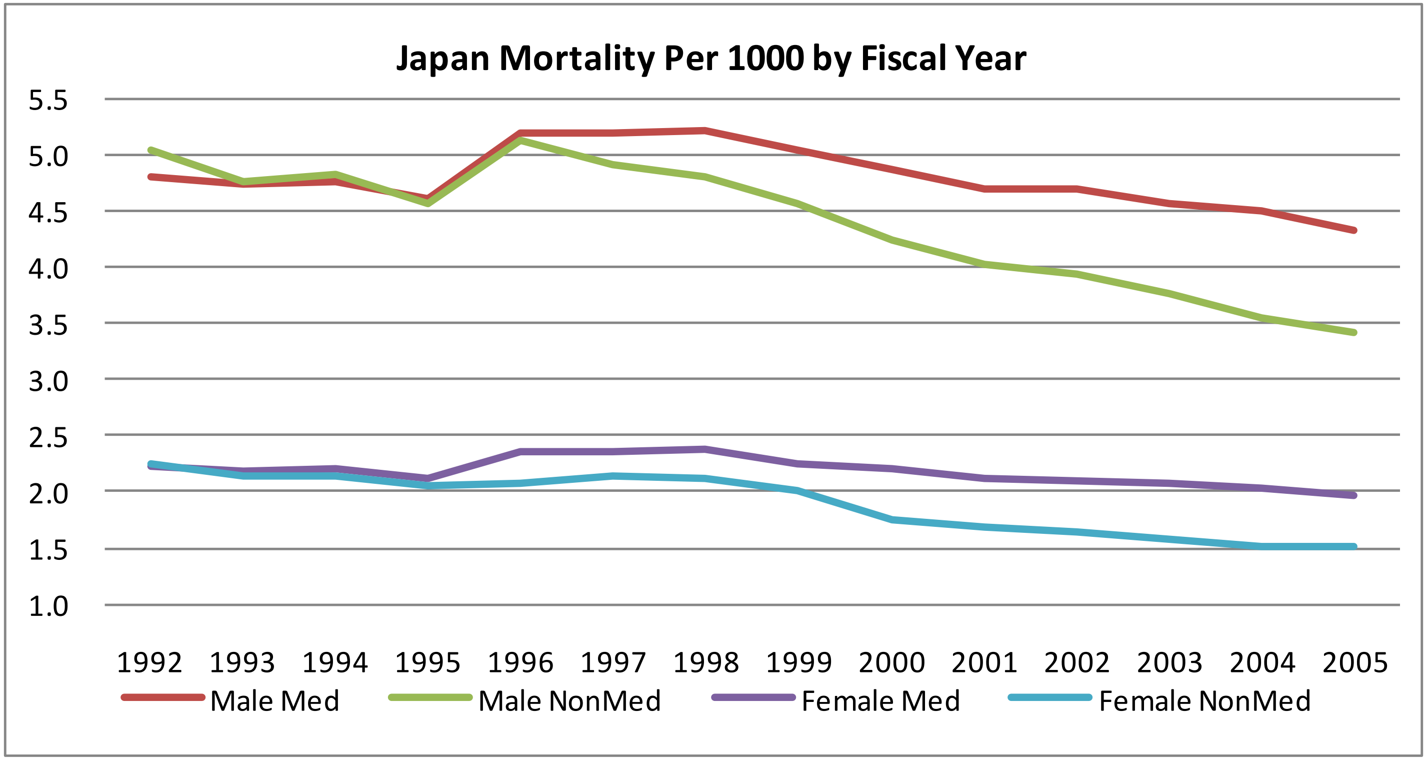


Cause of Death - Taiwan

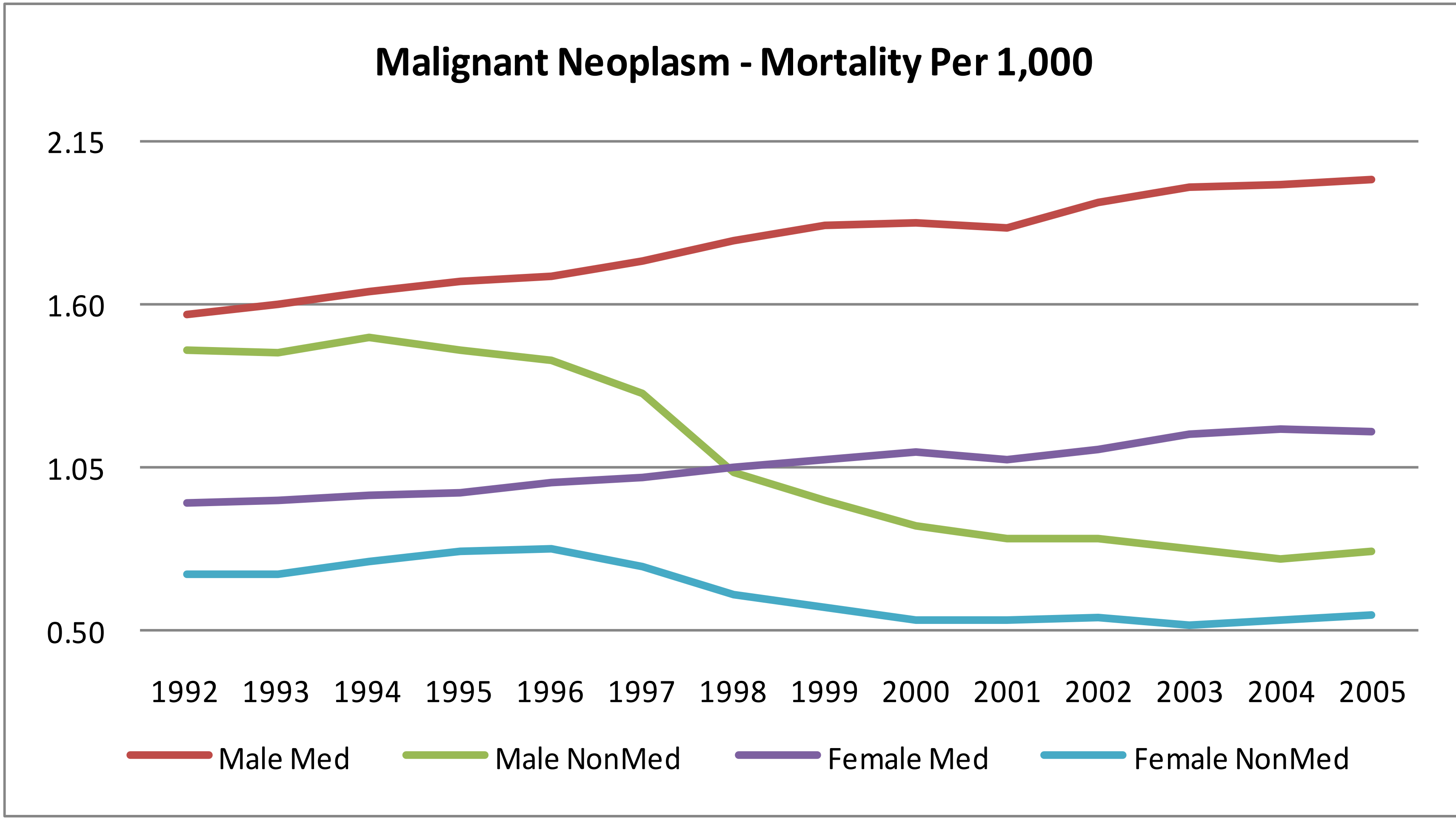
Taiwanese 5 leading Cause of Death

Cause	Male	Female	Both
Malignant Neoplasm	24.93	30.51	26.55
Accident	26.15	20.15	24.41
Circulatory	10.23	10.24	10.23
Digestive System	8.95	4.99	7.80
Mental and Nervous	7.97	7.12	7.72
Subtotal	78.23	73.01	76.72

Mortality Trend – Japan



Japan Mortality – Cause of Death



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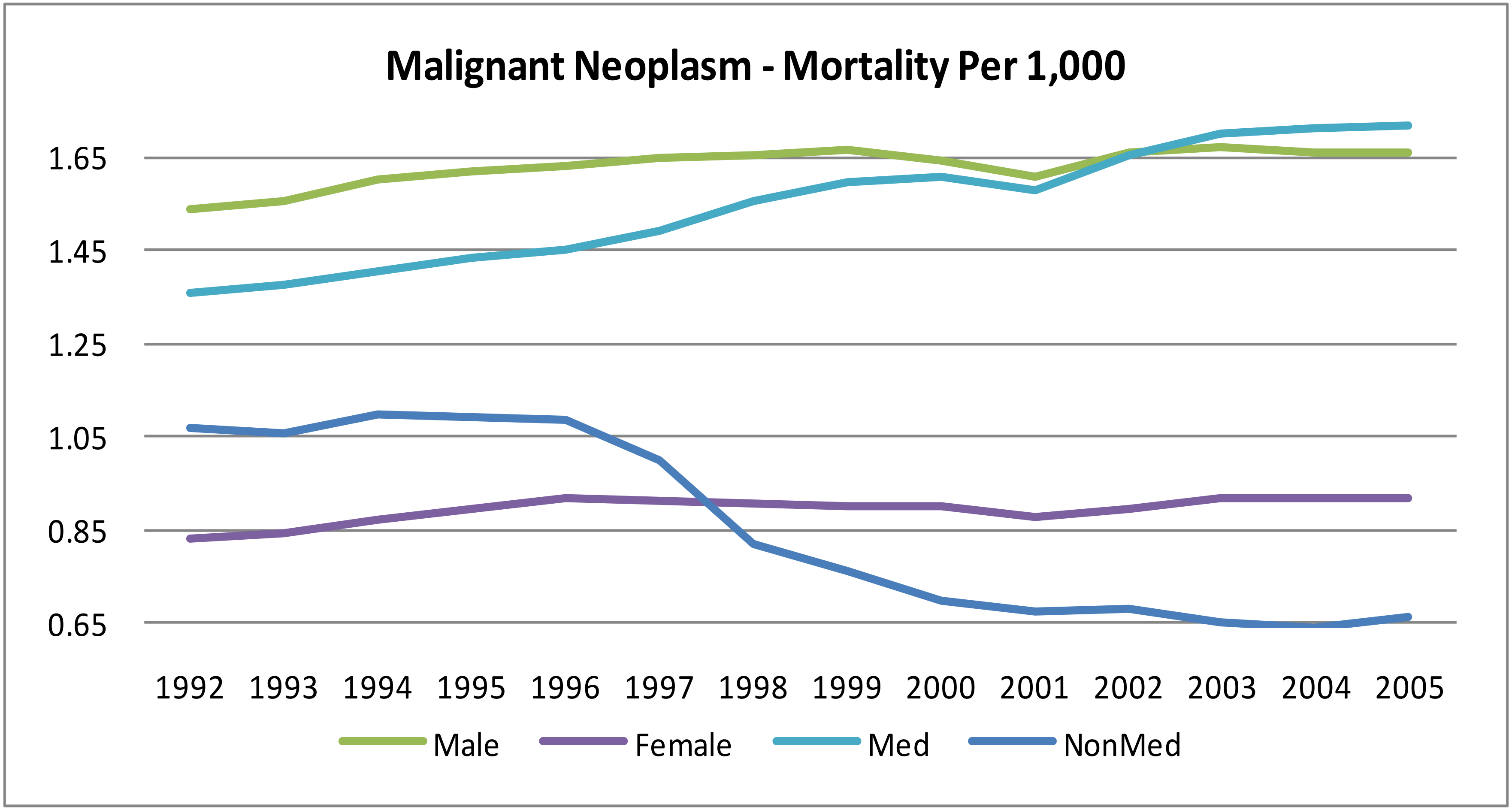

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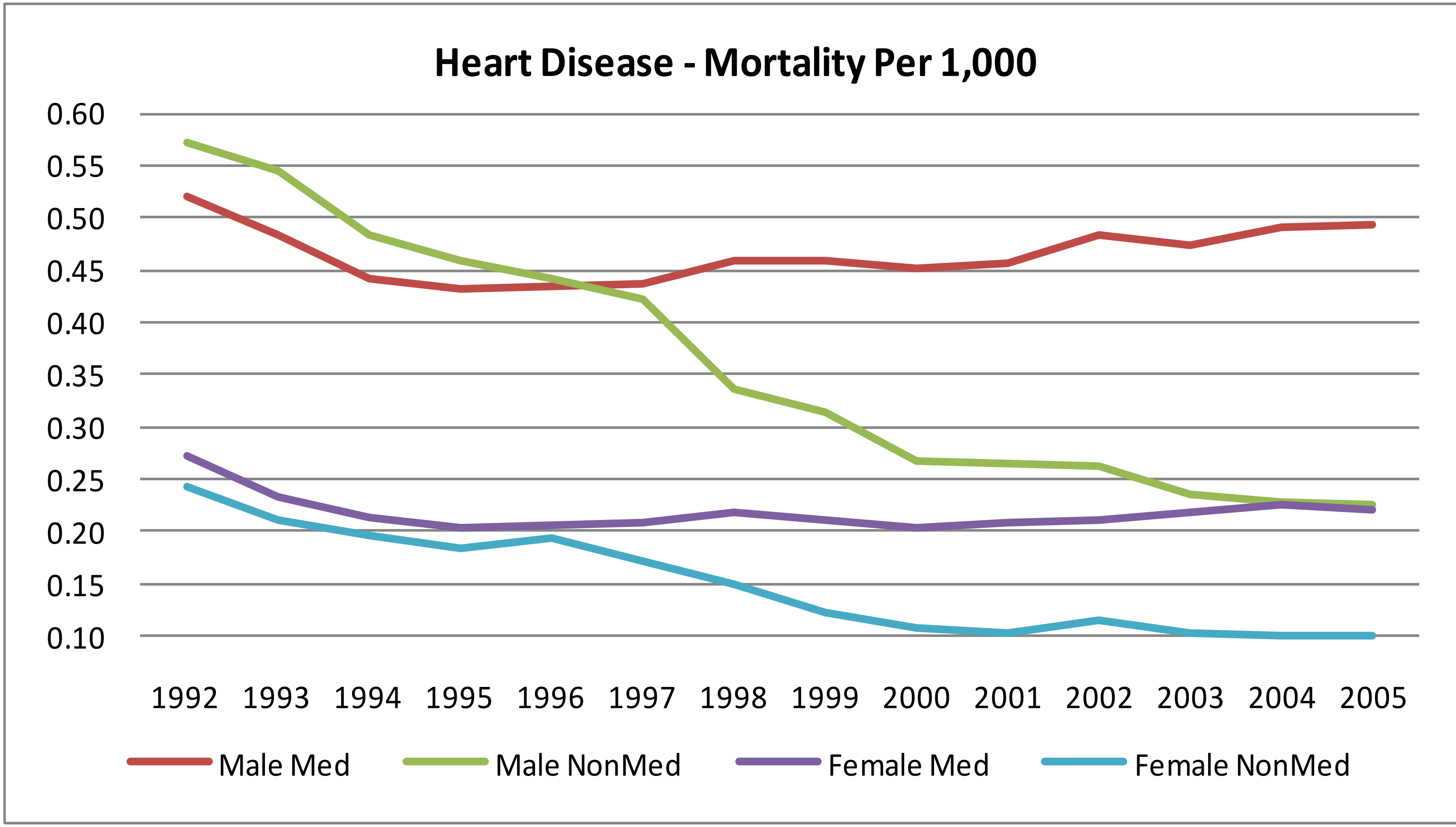

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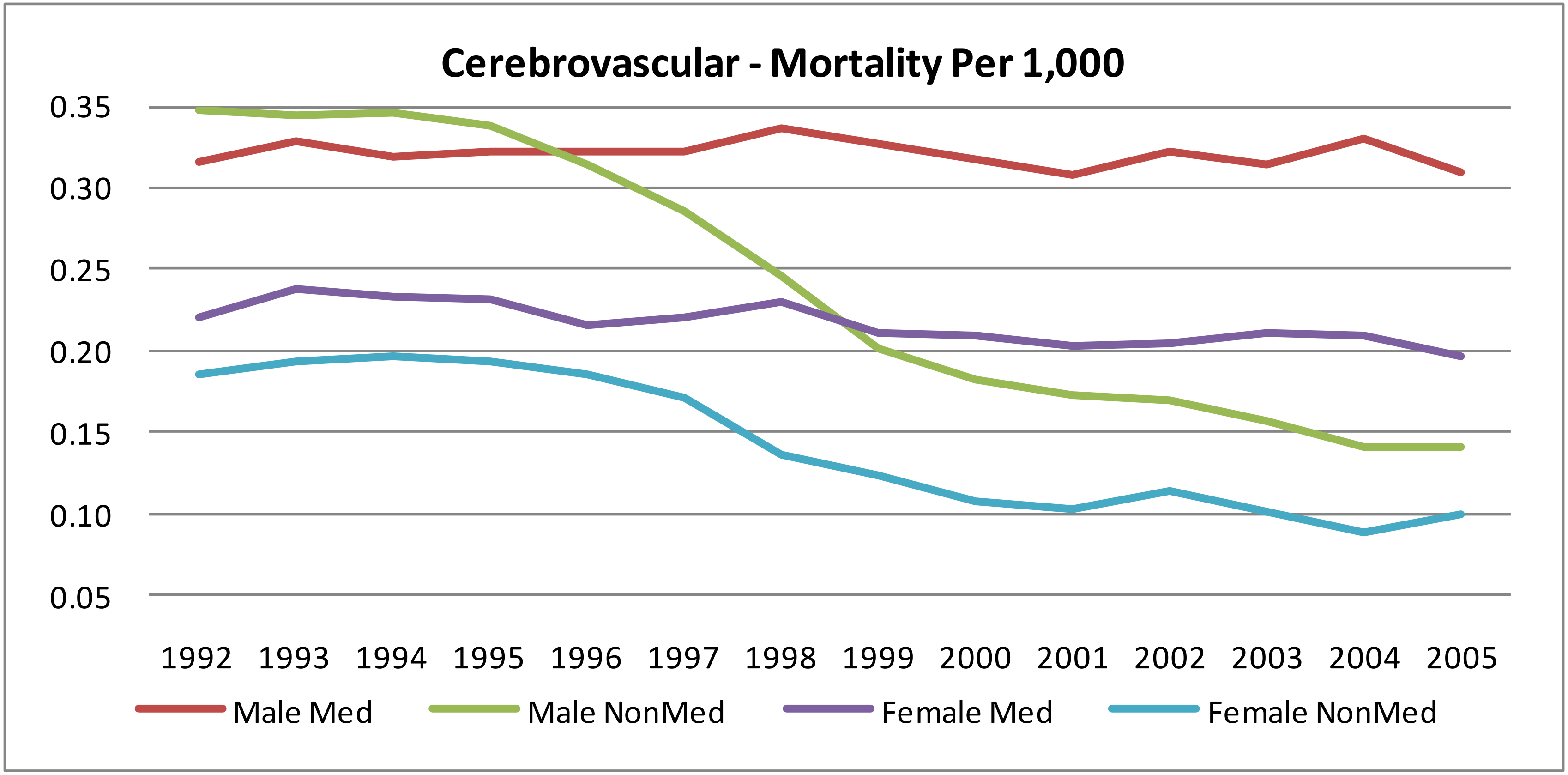

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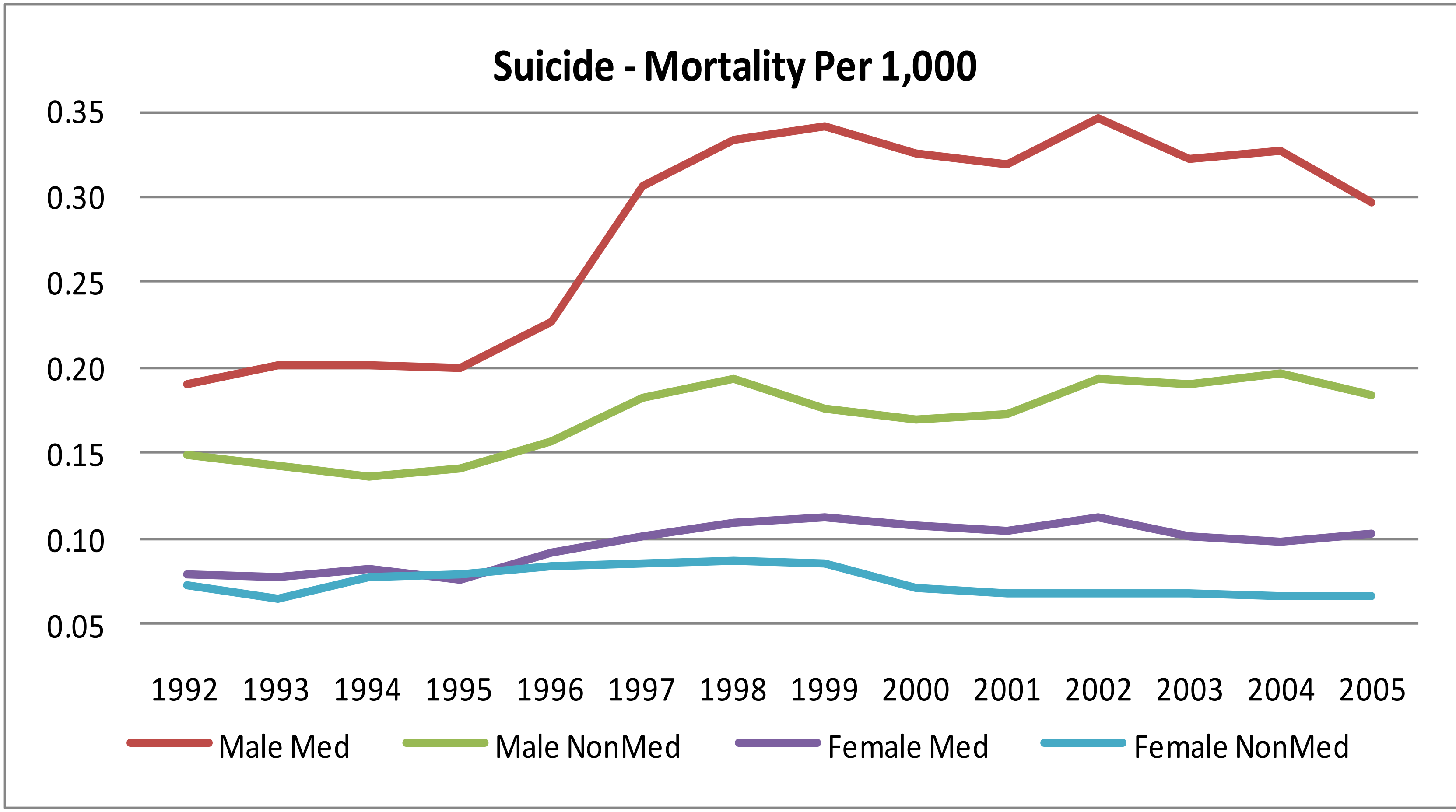
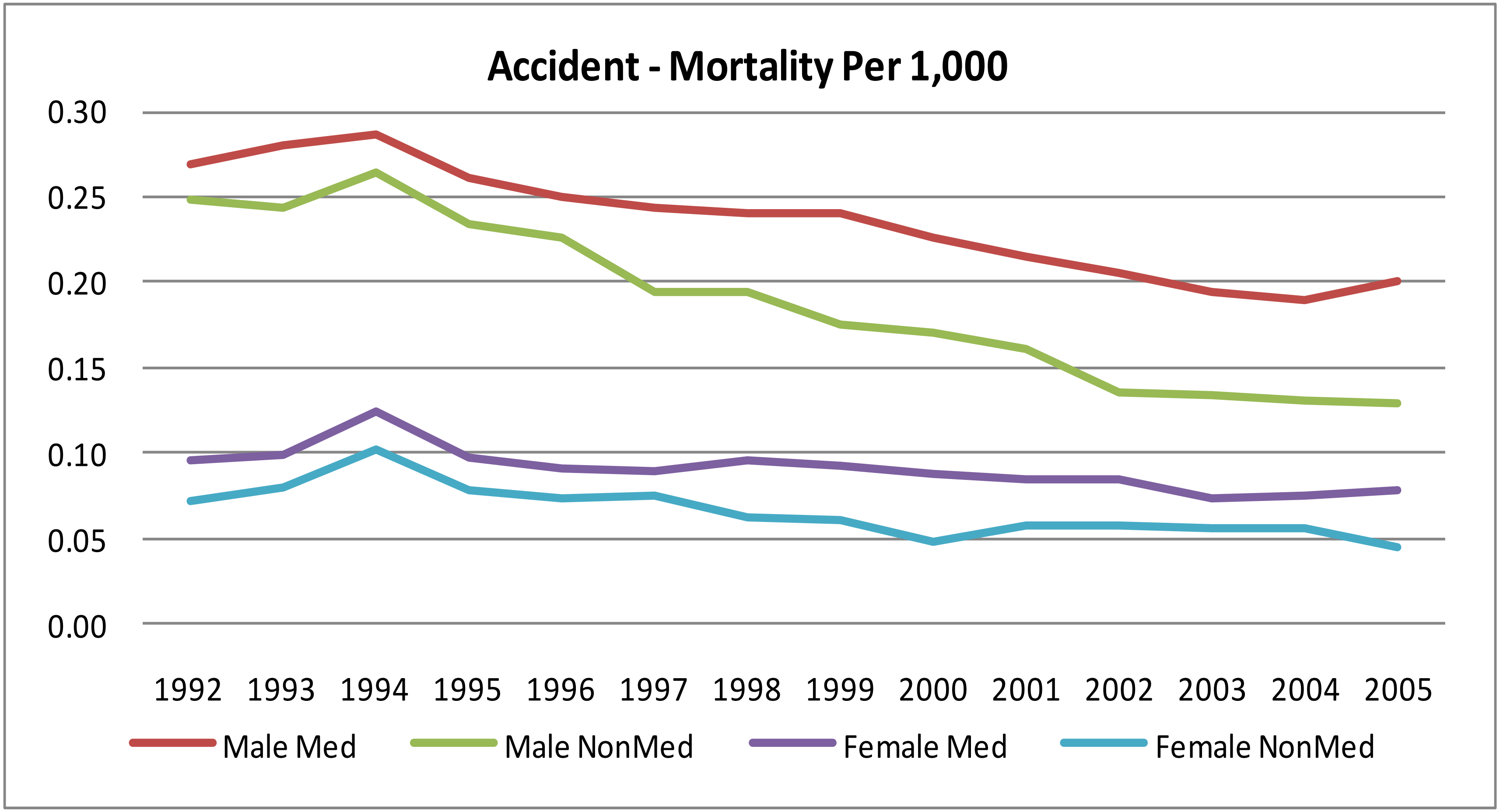


Japan Mortality – Cause of Death



Mortality Trend – Japan

Improvement/Deteriorate (+/−) Percent

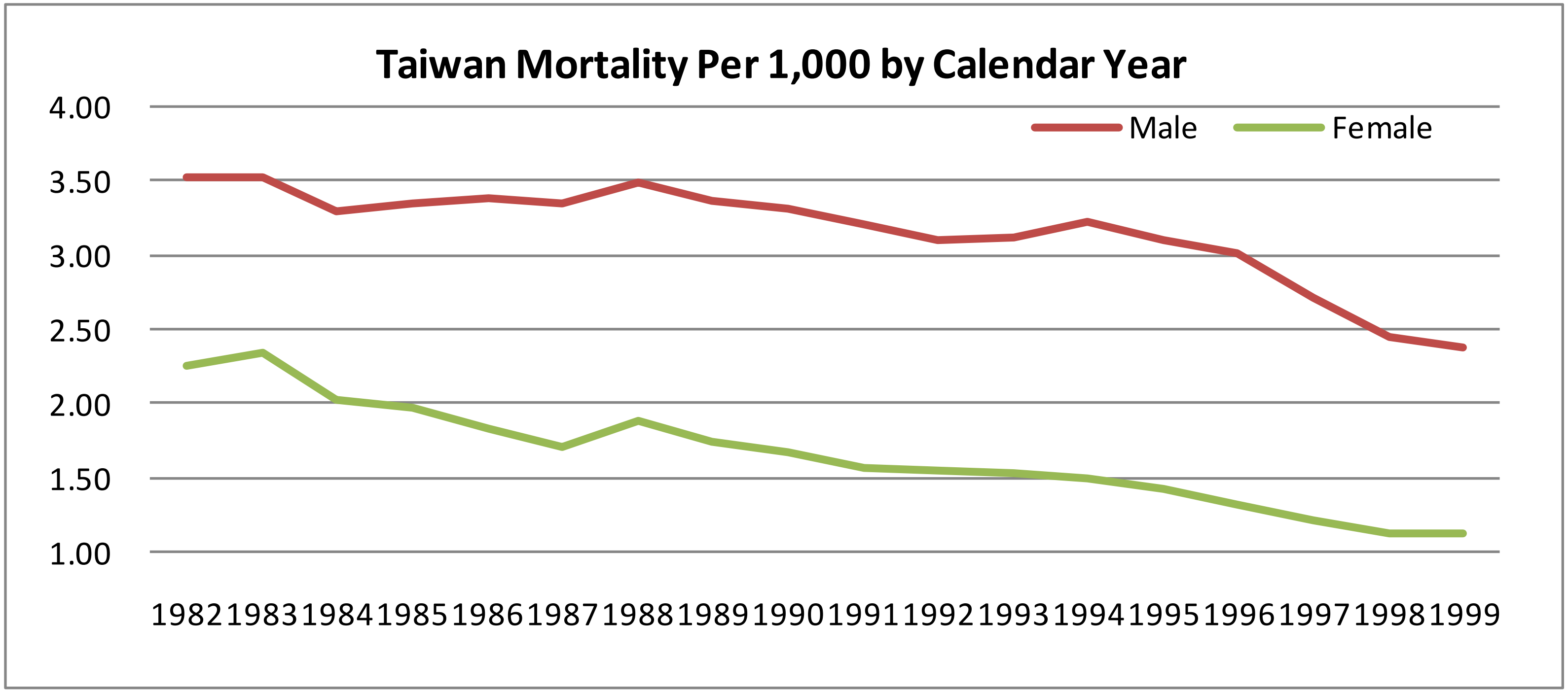
1997-05	Male		Female		Combined	
Cause	Med	NonMed	Med	NonMed	Male	Female
M.N.	(1.91)	5.79	(1.85)	2.80	(0.06)	(0.04)
H.D.	(1.61)	6.91	(0.75)	5.92	0.48	1.67
C.D.	0.32	8.07	1.26	5.67	2.22	3.18
S.C.	(0.07)	(0.28)	(0.34)	2.90	0.42	1.03
A.D.	2.44	4.78	1.53	5.34	3.18	3.26
Total	(1.63)	5.10	0.94	2.33	0.06	0.94

Mortality Trend – Korea

Korea EMT Improvement Factor

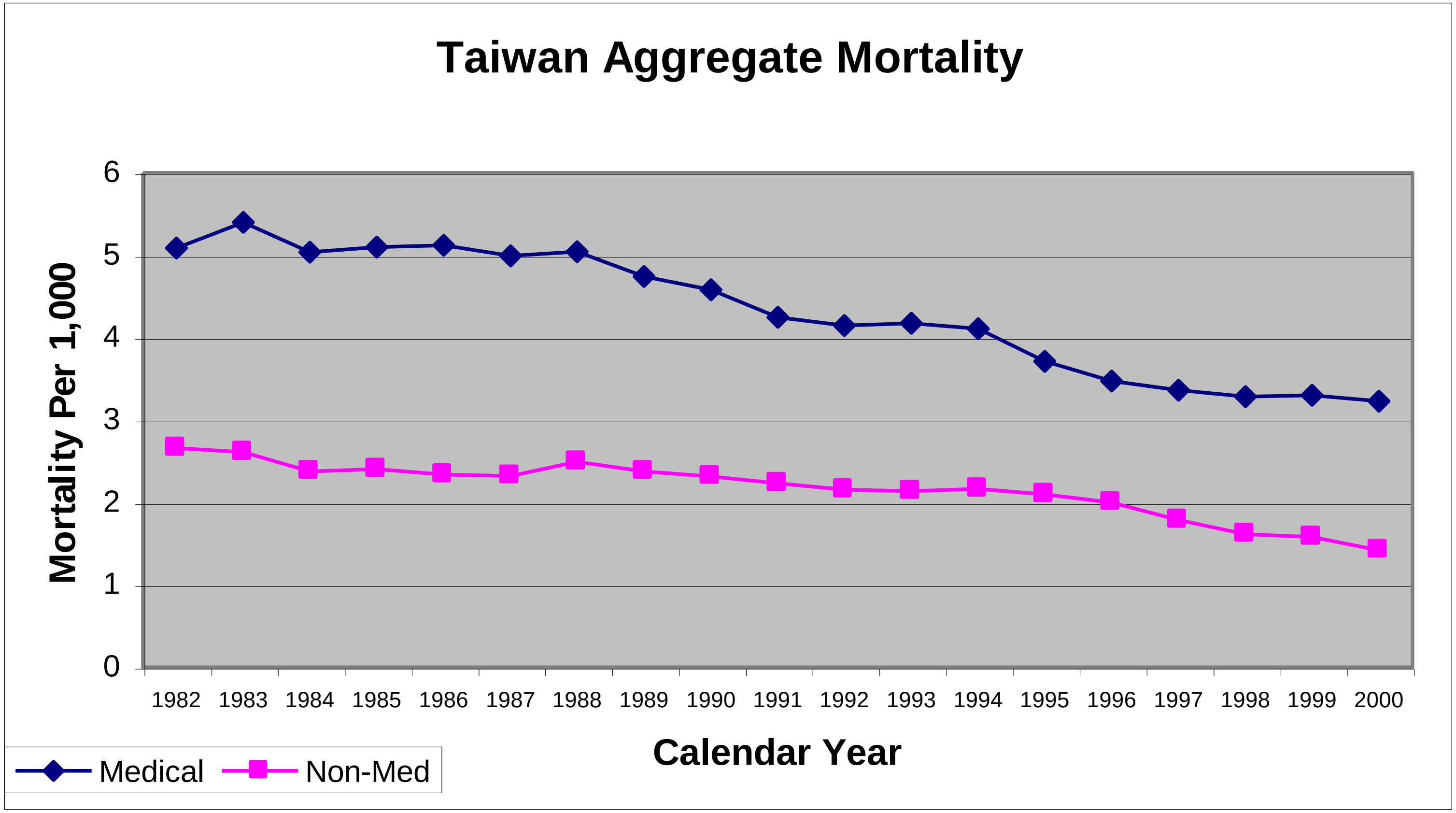
Period		Factor Percent			Korea EMT Mortality Per 1,000			
From	To	Male	Female		EMT	Period	Male	Female
3rd EMT	4thEMT	4.88	6.36		3rd	1988-1992	2.972	1.624
4th EMT	5th EMT	1.14	1.08		4th	1996-1998	2.093	1.025
5th EMT	6th EMT	1.25	4.21		5th	2000-2002	2.000	0.982
4th EMT	6th EMT	1.19	2.43		6th	2003-2005	1.926	0.863

Mortality Trend –Taiwan



Mortality Trend – Taiwan

Figure2 Taiwanese Aggregate Mortality by Type of Underwriting



Questions??